

A new class of solutions of anisotropic charged distributions on pseudo-spheroidal spacetime

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ABSTRACT

In the present article a new class of exact solutions of Einstein's field equations for charged anisotropic distribution is obtained on the background of pseudo-spheroidal spacetime characterized by the metric potential $g_{rr} = \frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}$, where K and R are geometric parameters of the spacetime. The radial pressure p_r and electric field intensity E are taken in the form $8\pi p_r = \frac{K-1}{R^2} \frac{\left(1-\frac{r^2}{R^2}\right)}{\left(1+K\frac{r^2}{R^2}\right)^2}$ and $E^2 = \frac{\alpha(K-1)\frac{r^2}{R^2}}{R^2\left(1+K\frac{r^2}{R^2}\right)^2}$. The bounds of geometric parameter K and the parameter α appearing in the expression of E^2 are obtained by imposing the requirements for a physically acceptable model. It is found that the model is in good agreement with the observational data of number of compact stars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3 given by Gangopadhyay *et al* [Gangopadhyay T., Ray S., Li X-D., Dey J. and Dey M., *Mon. Not. R. Astron. Soc.* **431** (2013) 3216]. When $\alpha = 0$, the model reduces to the uncharged anisotropic distribution given by Ratanpal *et al.* [Ratanpal B. S., Thomas V. O. and Pandya D. M., arXiv:1506.08512 [gr-qc](2015)]

Subject headings: General relativity; Exact solutions; Anisotropy; Relativistic compact stars; Charged distribution

1. Introduction

Mathematical model for generating superdense compact star models compatible with observational data has got wide attention among researchers. A number of papers have been appeared in literature in the recent past along this direction considering matter distribution incorporating charge (Maurya and Gupta 2011a,b,c; Pant and Maurya 2012; Maurya *et al* 2015). It has been suggested, as a result of theoretical investigations of Ruderman (1972) and Canuto (1974), that matter may not be isotropic in high density regime of 10^{15} gm/cm^3 . Hence it is pertinent to construct charged distribution incorporating anisotropy in pressure. Bonner (1960, 1965) has shown that a spherical distribution of matter can retain its equilibrium by counter balancing the gravitational force of attraction by Coulombian force of repulsion due to the presence of charge. It was shown by Stettner (1973) that a spherical distribution of uniform density accompanied by charge is more stable than distribution without charge. The study of charge distributions on spheroidal spacetimes have been carried out by Patel and Kopper (1987), Tikekar and Singh (1998), Sharma *et al* (2001), Gupta and Kumar (2005), Komathiraj and Maharaj (2007). The spheroidal spacetime is found to accommodate superdense stars like neutron stars in both charged and uncharged cases. Study of strange stars and quark stars in the presence of electric charge have been done by Sharma *et al* (2006), Sharma and Mukherjee (2001), Sharma and Mukherjee (2002). Recently charged fluid models have also been studied by Maurya and Gupta (2011a,b,c), Pant and Maurya (2012) & Maurya *et al* (2015).

In this paper, we have obtained a new class of solutions for charged fluid distribution on the background of pseudo spheroidal spacetime. Particular choices for radial pressure p_r and electric field intensity E are taken so that the physical requirements and regularity conditions are not violated. The bounds for the geometric parameter K and the parameter α associated with charge, are determined using various physical requirements that are expected to satisfy in its region of validity. It is found that these models can accommodate a number of pulsars like 4U 1820-30,

PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3, given by Gangopadhyay *et al* (2013). When $\alpha = 0$, the model reduces to the uncharged anisotropic distribution given by Ratanpal *et al* (2015).

In section 2, we have solved the field equations and in section 3, we have obtained the bounds for different parameters using physical acceptability and regularity conditions. In section 4, We have displayed a variety of pulsars in agreement wit the charged pseudo-spheroidal model developed. In particular we have studied a model for various physical conditions throughout the distribution and discussed the main results at the end of this section.

2. Spacetime Metric

We shall take the interior spacetime metric representing charged anisotropic matter distribution as

$$ds^2 = e^{\nu(r)} dt^2 - \left(\frac{1 + K \frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}} \right) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where K and R are geometric parameters and $K > 1$. This spacetime, known as pseudo-spheroidal spacetime, has been studied by number of researchers Tikekar and Thomas (1998, 1999, 2005); Thomas *et al* (2005); Thomas and Ratanpal (2007); Paul *et al* (2011); Paul and Chattopadhyay (2010); Chattopadhyay and Paul (2012) have found that it can accommodate compact superdense stars.

Since the metric potential g_{rr} is chosen apriori, the other metric potential $\nu(r)$ is to be determined by solving the Einstein-Maxwell field equations

$$R_i^j - \frac{1}{2} R \delta_i^j = 8\pi \left(T_i^j + \pi_i^j + E_i^j \right), \quad (2)$$

where,

$$T_i^j = (\rho + p) u_i u^j - p \delta_i^j, \quad (3)$$

$$\pi_i^j = \sqrt{3}S \left[c_i c^j - \frac{1}{2} (u_i u^j - \delta_i^j) \right], \quad (4)$$

and

$$E_i^j = \frac{1}{4\pi} \left(-F_{ik} F^{jk} + \frac{1}{4} F_{mn} F^{mn} \delta_i^j \right). \quad (5)$$

Here ρ , p , u_i , S and c^i , respectively, denote the proper density, fluid pressure, unit-four velocity, magnitude of anisotropic tensor and a radial vector given by $(0, -e^{-\lambda/2}, 0, 0)$. F_{ij} denotes the anti-symmetric electromagnetic field strength tensor defined by

$$F_{ij} = \frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j}, \quad (6)$$

which satisfies the Maxwell equations

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0, \quad (7)$$

and

$$\frac{\partial}{\partial x^k} (F^{ik} \sqrt{-g}) = 4\pi \sqrt{-g} J^i, \quad (8)$$

where g denotes the determinant of g_{ij} , $A_i = (\phi(r), 0, 0, 0)$ is four-potential and

$$J^i = \sigma u^i, \quad (9)$$

is the four-current vector where σ denotes the charge density.

The only non-vanishing components of F_{ij} is $F_{01} = -F_{10}$. Here

$$F_{01} = -\frac{e^{\frac{\nu+\lambda}{2}}}{r^2} \int_0^r 4\pi r^2 \sigma e^{\lambda/2} dr, \quad (10)$$

and the total charge inside a radius r is given by

$$q(r) = 4\pi \int_0^r \sigma r^2 e^{\lambda/2} dr. \quad (11)$$

The electric field intensity E can be obtained from $E^2 = -F_{01} F^{01}$, which subsequently reduces to

$$E = \frac{q(r)}{r^2}. \quad (12)$$

The field equations given by (2) are now equivalent to the following set of the non-linear ODE's

$$\frac{1 - e^{-\lambda}}{r^2} + \frac{e^{-\lambda} \lambda'}{r} = 8\pi\rho + E^2, \quad (13)$$

$$\frac{e^{-\lambda} - 1}{r^2} + \frac{e^{-\lambda} v'}{r} = 8\pi p_r - E^2, \quad (14)$$

$$e^{-\lambda} \left(\frac{v''}{2} + \frac{v'^2}{4} - \frac{v' \lambda'}{4} + \frac{v' - \lambda'}{2r} \right) = 8\pi p_{\perp} + E^2, \quad (15)$$

where we have taken

$$p_r = p + \frac{2S}{\sqrt{3}}, \quad (16)$$

$$p_{\perp} = p - \frac{S}{\sqrt{3}}. \quad (17)$$

Because $e^{\lambda} = \frac{1+K\frac{r^2}{R^2}}{1+\frac{r^2}{R^2}}$, the metric potential λ is known function of r . The set of equations (13) - (15) are to be solved for five unknowns v , ρ , p_r , p_{\perp} and E . So we have two free variables for which suitable assumption can be made. We shall assume the following expressions for p_r and E .

$$8\pi p_r = \frac{K-1}{R^2} \frac{1 - \frac{r^2}{R^2}}{\left(1 + K\frac{r^2}{R^2}\right)^2}, \quad (18)$$

$$E^2 = \frac{\alpha(K-1)}{R^2} \frac{\frac{r^2}{R^2}}{\left(1 + K\frac{r^2}{R^2}\right)}. \quad (19)$$

It can be noticed from equation (18) that p_r vanishes at $r = R$ and hence we take the geometric parameter R as the radius of distribution. Further $p_r \geq 0$ for all values of r in the range $0 \leq r \leq R$. It can also be noted that E^2 is regular at $r = 0$. On substituting the values of p_r and E^2 in (14) we obtain, after a lengthy calculation

$$e^v = CR^{\frac{[K^2 - (2+\alpha)K + \alpha + 1]}{K}} \left(1 + K\frac{r^2}{R^2}\right)^{\left(\frac{K+\alpha+1}{2K}\right)} \left(1 + \frac{r^2}{R^2}\right)^{\frac{K-\alpha-3}{2}}, \quad (20)$$

where C is a constant of integration. Hence, the spacetime metric takes the explicit form

$$ds^2 = CR^{\frac{[K^2-(2+\alpha)K+\alpha+1]}{K}} \left(1 + K \frac{r^2}{R^2}\right)^{\left(\frac{K+\alpha+1}{2K}\right)} \left(1 + \frac{r^2}{R^2}\right)^{\frac{K-\alpha-3}{2}} dt^2 - \left(\frac{1 + K \frac{r^2}{R^2}}{1 + \frac{r^2}{R^2}}\right) dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (21)$$

The constant of integration C can be evaluated by matching the interior spacetime metric with Riessner-Nordström metric

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right) dt^2 - \left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (22)$$

across the boundary $r = R$. This gives

$$M = \frac{R}{2} \frac{[K^2 + \alpha(K-1) - 1]}{(1+K)^2}, \quad (23)$$

and

$$C = R^{-\frac{[K^2-(2+\alpha)K+\alpha+1]}{K}} (1+K)^{-\left(\frac{3K+\alpha+1}{2K}\right)} 2^{\left(\frac{\alpha-K+5}{2}\right)}. \quad (24)$$

Here M denotes the total mass of the charged anisotropic distribution.

3. Physical Requirements and Bounds for Parameters

The gradient of radial pressure is obtained from equation (18) in the form

$$8\pi \frac{dp_r}{dr} = -\frac{2r(K-1)}{R^4} \frac{1+2K-K\frac{r^2}{R^2}}{\left(1+K\frac{r^2}{R^2}\right)^3} < 0. \quad (25)$$

It can be noticed from equation (25) that the radial pressure is decreasing function of r . Now, equation (13) gives the density of the distribution as

$$8\pi\rho = \left(\frac{K-1}{R^2}\right) \frac{3+(K-\alpha)\frac{r^2}{R^2}}{\left(1+K\frac{r^2}{R^2}\right)^2}. \quad (26)$$

The condition $\rho(r=0) > 0$ is clearly satisfied and $\rho(r=R) > 0$ gives the following inequality connecting α and K .

$$0 \leq \alpha < 3 + K. \quad (27)$$

Differentiating (26) with respect to r , we get

$$8\pi \frac{d\rho}{dr} = -\frac{2r(K-1)}{R^4} \frac{5K + \alpha + K(K-\alpha)\frac{r^2}{R^2}}{\left(1 + K\frac{r^2}{R^2}\right)^3}. \quad (28)$$

It is observed that $\frac{d\rho}{dr}(r=0) = 0$ and $\frac{d\rho}{dr}(r=R) < 0$ leads to the inequality

$$K^2 - K(\alpha - 5) + \alpha \geq 0. \quad (29)$$

The inequality (29) together with the condition $K > 1$ give a bound for α as

$$0 \leq \alpha < \frac{K(K+5)}{K-1}. \quad (30)$$

The expression for p_{\perp} is

$$8\pi p_{\perp} = \frac{4K - 4 + X_1 \frac{r^2}{R^2} + X_2 \frac{r^4}{R^4} + X_3 \frac{r^6}{R^6}}{R^2 \left(4 + Y_1 \frac{r^2}{R^2} + Y_2 \frac{r^4}{R^4} + Y_3 \frac{r^6}{R^6} + 4K^3 \frac{r^8}{R^8}\right)}, \quad (31)$$

where, $X_1 = 4K^2 + (-12\alpha - 16)K + 12\alpha + 12$, $X_2 = 6K^3 + (-10\alpha - 22)K^2 + (4\alpha + 14)K + 6\alpha + 2$, $X_3 = K^4 + (-2\alpha - 4)K^3 + (\alpha^2 + 2\alpha + 6)K^2 + (-2\alpha^2 - 2\alpha - 4)K + \alpha^2 + 2\alpha + 1$, $Y_1 = 12K + 4$, $Y_2 = 12K^2 + 12K$ and $Y_3 = 4K^3 + 12K^2$.

The condition $p_{\perp} > 0$ at the boundary $r = R$ imposes a restriction on K and α respectively given by

$$K > 2\sqrt{3} - 1 \quad (32)$$

and

$$0 \leq \alpha < \frac{10 + 5K + K^2}{K - 1} - \sqrt{\frac{89 + 102K + 57K^2 + 8K^3}{(K - 1)^2}}. \quad (33)$$

The expression for $\frac{dp_{\perp}}{dr}$ is given by

$$\frac{dp_{\perp}}{dr} = \frac{-r \left(8K^2 + (12\alpha + 8)K - 12\alpha - 16 + A_1 \frac{r^2}{R^2} + A_2 \frac{r^4}{R^4} + A_3 \frac{r^6}{R^6} + A_4 \frac{r^8}{R^8} \right)}{R^4 \left(2 + B_1 \frac{r^2}{R^2} + B_2 \frac{r^4}{R^4} + B_3 \frac{r^6}{R^6} + B_4 \frac{r^8}{R^8} + B_5 \frac{r^{10}}{R^{10}} + 2K^4 \frac{r^{12}}{R^{12}} \right)}, \quad (34)$$

where, $A_1 = -4K^3 + (28 - 4\alpha)K^2 + (16\alpha - 20)K - 12\alpha - 4$, $A_2 = 3K^4 + (-4\alpha - 4)K^3 + (-3\alpha^2 - 28\alpha - 30)K^2 + (6\alpha^2 + 44\alpha + 36)K - 3\alpha^2 - 12\alpha - 5$, $A_3 = 10K^4 + (-16\alpha - 36)K^3 + (-2\alpha^2 + 4\alpha + 16)K^2 + (4\alpha^2 + 16\alpha + 12)K - 2\alpha^2 - 4\alpha - 2$, $A_4 = K^5 + (-2\alpha - 4)K^4 + (\alpha^2 + 2\alpha + 6)K^3 + (-2\alpha^2 - 2\alpha - 4)K^2 + \alpha^2 + 2\alpha + 1$, $B_1 = 8k + 4$, $B_2 = 12K^2 + 16K + 2$, $B_3 = 8K^3 + 24K^2 + 8K$, $2K^4 + 16K^3 + 12K^2$ and $B_4 = 4K^4 + 8K^3$.

The value of $\frac{dp_{\perp}}{dr} = 0$ at the origin and $\frac{dp_{\perp}}{dr}(r = R) < 0$ gives the following bounds for K and α respectively

$$2\sqrt{13} - 5 < K < 5 \quad (35)$$

and

$$0 \leq \alpha < \frac{K^3 + 10K^2 + 25K - 20}{K^2 - 6K + 5} + \sqrt{\frac{16K^5 + 233K^4 + 252K^3 + 278K^2 - 788K + 265}{(K^2 - 6K + 5)^2}} \quad (36)$$

In order to examine the strong energy condition, we evaluate the expression $\rho - p_r - 2p_{\perp}$ at the centre and on the boundary of the star. It is found that

$$(\rho - p_r - 2p_{\perp})(r = 0) = 0, \quad (37)$$

and $(\rho - p_r - 2p_{\perp})(r = R) > 0$ gives the bound on K and α , namely

$$1 < K < 1 + 2\sqrt{6} \quad (38)$$

$$0 \leq \alpha < \frac{8 + 3K + K^2}{K - 1} + \sqrt{\frac{41 + 46K + 49K^2 + 8K^3}{(K - 1)^2}}. \quad (39)$$

The expressions for adiabatic sound speed $\frac{dp_r}{d\rho}$ and $\frac{dp_{\perp}}{d\rho}$ in the radial and transverse directions, respectively, are given by

$$\frac{dp_r}{d\rho} = \frac{1 + 2K - K \frac{r^2}{R^2}}{5k + \alpha + K(K - \alpha) \frac{r^2}{R^2}}, \quad (40)$$

and

$$\frac{dp_{\perp}}{d\rho} = \frac{\left(1 + K \frac{r^2}{R^2}\right)^3 \left[8K^2 + (12\alpha + 8)K - 12\alpha - 16 + C_1 \frac{r^2}{R^2} + C_2 \frac{r^4}{R^4} + C_3 \frac{r^6}{R^6} + C_4 \frac{r^8}{R^8}\right]}{2(K-1) \left[5K + \alpha + K(K-\alpha) \frac{r^2}{R^2}\right] \left[2 + D_1 \frac{r^2}{R^2} + D_2 \frac{r^4}{R^4} + D_3 \frac{r^6}{R^6} + D_4 \frac{r^8}{R^8} + D_5 \frac{r^{10}}{R^{10}} + 2K^4 \frac{r^{12}}{R^{12}}\right]}, \quad (41)$$

where, $C_1 = -4K^3 + (18 - 4\alpha)K^2 + (16\alpha - 20)K - 12\alpha - 4$, $C_2 = 3K^4 + (-4\alpha - 4)K^3 + (-3\alpha^2 - 28\alpha - 30)K^2 + (6\alpha^2 + 44\alpha + 36)K - 3\alpha^2 - 12\alpha - 5$, $C_3 = 10K^4 + (-16\alpha - 36)K^3 + (-2\alpha^2 + 4\alpha + 16)K^2 + (4\alpha^2 + 16\alpha + 12)K - 2\alpha^2 - 4\alpha - 2$, $C_4 = K^5 + (-2\alpha - 4)K^4 + (\alpha^2 + 2\alpha + 6)K^3 + (-2\alpha^2 - 2\alpha - 4)K^2 + (\alpha^2 + 2\alpha + 1)K$, $D_1 = 8K + 4$, $D_2 = 12K^2 + 16K + 2$, $D_3 = 8K^3 + 24K^2 + 8K$, $D_4 = 2K^4 + 16K^3 + 12K^2$ and $D_5 = 4K^4 + 8K^3$.

The condition $0 \leq \frac{dp_r}{d\rho} \leq 1$ is evidently satisfied at the centre whereas at the boundary it gives a restriction on α as

$$0 \leq \alpha < \frac{K^2 + 4K - 1}{K - 1}, \quad K > 1. \quad (42)$$

Further $\frac{dp_{\perp}}{d\rho} \leq 1$ at the centre will lead to the following inequalities

$$K > \frac{4}{3} \quad (43)$$

and

$$0 \leq \alpha < \frac{1}{2}(3K - 4). \quad (44)$$

Moreover at the boundary ($r = R$), we have the following restrictions on K and α .

$$-5 + 2\sqrt{13} \leq K < 5 \quad (45)$$

and

$$0 \leq \alpha \leq \frac{K^3 + 10K^2 + 25K - 20}{K^2 - 6K + 5} + \sqrt{\frac{16K^5 + 233K^4 + 252K^3 + 278K^2 - 788K + 265}{(K^2 - 6K + 5)^2}}, \quad (46)$$

The necessary condition for the model to represent a stable relativistic star is that $\Gamma > \frac{4}{3}$ throughout the star. $\Gamma > \frac{4}{3}$ at $r = 0$ gives a bound on α which is identical to (27). Further, $\Gamma \rightarrow \infty$

as $r \rightarrow R$ and hence the condition is automatically satisfied. It can be noticed that $E = 0$ at $r = 0$, showing the regularity of the charged distribution.

The upper limits of α in the inequalities (27), (30), (33), (36), (39), (42) and (44) for different permissible values of K are shown in Table 1. It can be noticed that for $2.4641 < K \leq 3.7641$ the bound for α is $0 \leq \alpha \leq 0.6045$.

Table 1: The upper limits of α for different permissible values of K .

K	Inequality Numbers						
	(27)	(30)	(33)	(42)	(44)	(36)	(39)
2.4641	5.4641	12.5622	0.0000	10.1962	1.6962	0.0802	30.9893
2.5041	5.5041	12.4932	0.0170	10.1635	1.7562	0.0938	30.6186
2.6041	5.6041	12.3445	0.0599	10.0977	1.9062	0.1287	29.7861
2.7041	5.7041	12.2250	0.1036	10.0514	2.0562	0.1648	29.0693
2.8041	5.8041	12.1299	0.1480	10.0213	2.2062	0.2021	28.4488
2.9041	5.9041	12.0552	0.1931	10.0048	2.3562	0.2405	27.9094
3.0041	6.0041	11.9980	0.2388	10.0000	2.5062	0.2798	27.4388
3.1041	6.1041	11.9557	0.2852	10.0052	2.6562	0.3201	27.0271
3.2041	6.2041	11.9263	0.3321	10.0189	2.8062	0.3612	26.6662
3.3041	6.3041	11.9082	0.3795	10.0401	2.9562	0.4030	26.3495
3.4041	6.4041	11.8998	0.4275	10.0679	3.1062	0.4457	26.0714
3.5041	6.5041	11.9002	0.4760	10.1015	3.2562	0.4890	25.8272
3.6041	6.6041	11.9082	0.5251	10.1401	3.4062	0.5330	25.6130
3.7041	6.7041	11.9230	0.5745	10.1833	3.5562	0.5776	25.4254
3.7541	6.7541	11.9327	0.5995	10.2065	3.6312	0.6001	25.3407
3.7641	6.7641	11.9348	0.6045	10.2112	3.6462	0.6047	25.3244

4. Application to Compact Stars and Discussion

In order to compare the charged anisotropic model on pseudo-spheroidal spacetime with observational data, we have considered the pulsar PSR J1614-2230 whose estimated mass and radius are $1.97M_{\odot}$ and 9.69 km . On substituting these values in equation (23) we have obtained

the values of adjustable parameters K and α as $K = 3.58524$ and $\alpha = 0.292156$ respectively which are well inside their permitted limits. Similarly assuming the estimated masses and radii of some well known pulsars like 4U 1820-30, PSR J1903+327, 4U 1608-52, Vela X-1, PSR J1614-2230, Cen X-3, we have displayed the values of the parameters K and α , the central density ρ_c , surface density ρ_R , the compactification factor $u = \frac{M}{R}, \frac{dp_r}{d\rho}(r=0)$ and charge Q inside the star in Table 2. From the table it is clear that our model is in good agreement with the most recent observational data of pulsars given by Gangopadhyay *et al* (2013).

Table 2: Estimated physical values based on the observational data

STAR	K	M (M_\odot)	R (Km)	ρ_c (MeV fm^{-3})	ρ_R (MeV fm^{-3})	$u(= \frac{M}{R})$	$\left(\frac{dp_r}{d\rho}\right)_{r=0}$	Q Coulomb
4U 1820-30	2.815	1.58	9.1	1980.14	250.46	0.256	0.461	4.031×10^{20}
PSR J1903+327	2.880	1.667	9.438	1906.90	235.92	0.261	0.460	4.184×10^{20}
4U 1608-52	3.122	1.74	9.31	2212.22	252.97	0.276	0.455	4.127×10^{20}
Vela X-1	3.078	1.77	9.56	2054.99	238.25	0.273	0.456	4.240×10^{20}
PSR J1614-2230	3.585	1.97	9.69	2487.35	248.17	0.300	0.448	4.262×10^{20}
Cen X-3	2.589	1.49	9.178	1705.08	233.65	0.239	0.466	4.044×10^{20}

In order to examine the nature of physical quantities throughout the distribution, we have considered a particular star PSR J1614-2230, whose tabulated mass and radius are $M = 1.97M_\odot$, $R = 9.69 \text{ km}$. Choosing $K = 3.58524$ and $\alpha = 0.292156$, we have shown the variations of density and pressures in both the charged and uncharged cases in Figure 1, Figure 2 and Figure 3. It can be noticed that the pressure is decreasing radially outward. The density in the uncharged case is always greater than the density in the charged case. Similarly the radial pressure p_r and transverse pressure p_\perp are decreasing radially outward. Similar to that of density, p_r and p_\perp in the uncharged case accommodate more values compared to charged case.

The variation of anisotropy shown in Figure 4 is initially decreasing with negative values reaches a minimum and then increases. In this case also anisotropy takes lesser values in the charged case compared to uncharged case. The square of sound in the radial and transverse direction (i.e. $\frac{dp_r}{d\rho}$ and $\frac{dp_\perp}{d\rho}$) are shown in Figure 5 and Figure 6 respectively and found that they are less than 1. The graph of $\rho - p_r - 2p_\perp$ against radius is plotted Figure 7. It can be observed that it is non-negative for $0 \leq r \leq R$ and hence strong energy condition is satisfied throughout the star.

A necessary condition for the exact solution to represent stable relativistic star is that the relativistic adiabatic index given by $\Gamma = \frac{\rho + p_r}{p_r} \frac{dp_r}{d\rho}$ should be greater than $\frac{4}{3}$. The variation of adiabatic index throughout the star is shown in Figure 8 and it is found that $\Gamma > \frac{4}{3}$ throughout the distribution both in charged and uncharged case. Though we have not assumed any equation of state in the explicit form $p_r = p_r(\rho)$ and $p_\perp = p_\perp(\rho)$, we have shown the relation between p_r, p_\perp against ρ in the graphical form as displayed in Figure 9 and Figure 10. For a physically acceptable relativistic star the gravitational redshift must be positive and finite at the centre and on the boundary. Further it should be a decreasing function of r . Figure 11 shows that this is indeed the case. Finally we have plotted the graph of E^2 against r which is displayed in Figure 12. Initially E^2 increases from 0 and reaches a maximum values and then decreases radially outward. The model reduces to the uncharged anisotropic distribution given by Ratanpal *et al* (2015) when $\alpha = 0$.

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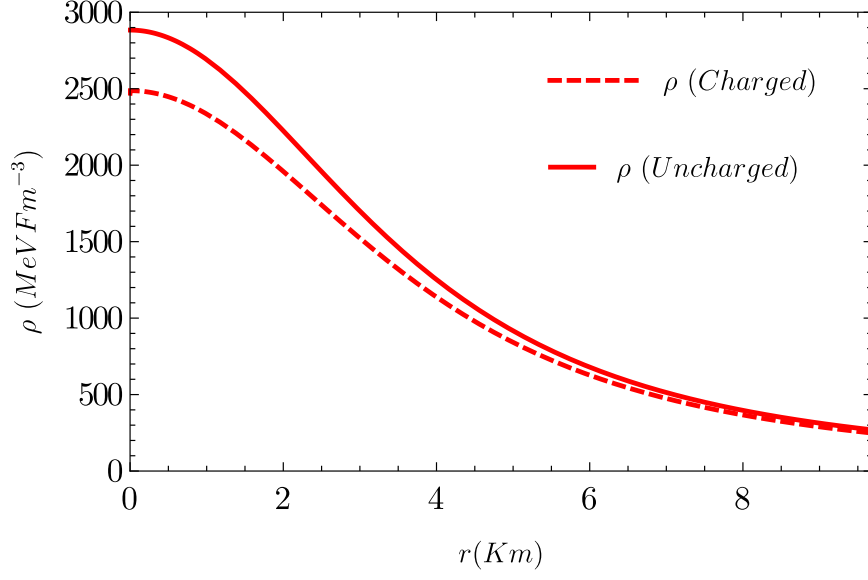


Fig. 1.— Variation of density against radial variable r .

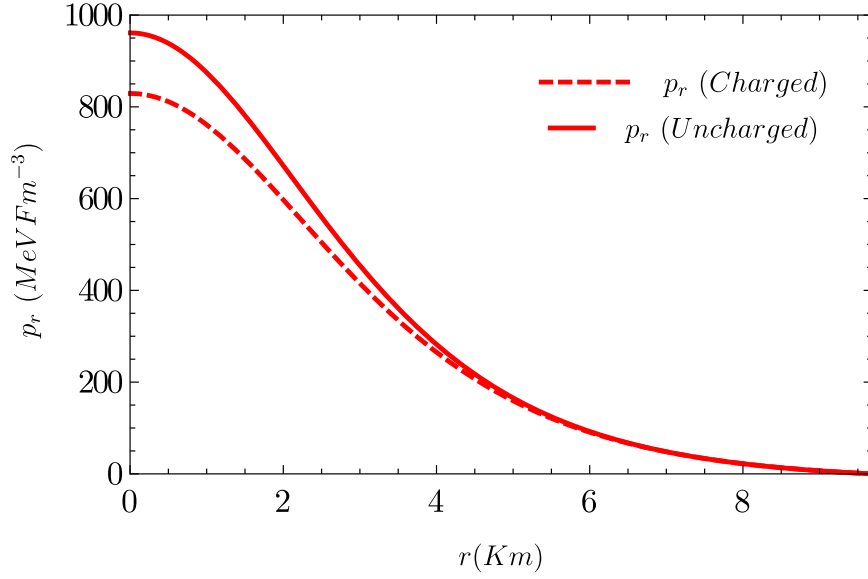


Fig. 2.— Variation of radial pressures against radial variable r .

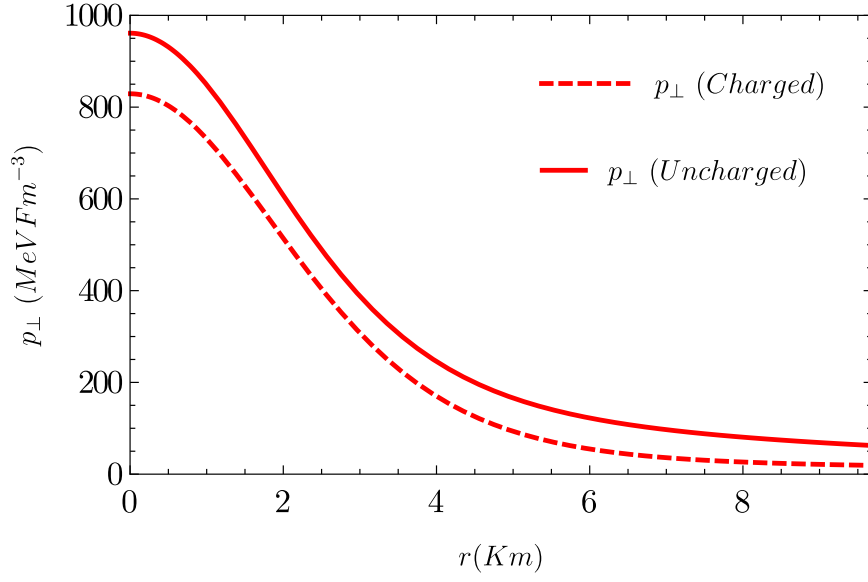


Fig. 3.— Variation of transverse pressures against radial variable r

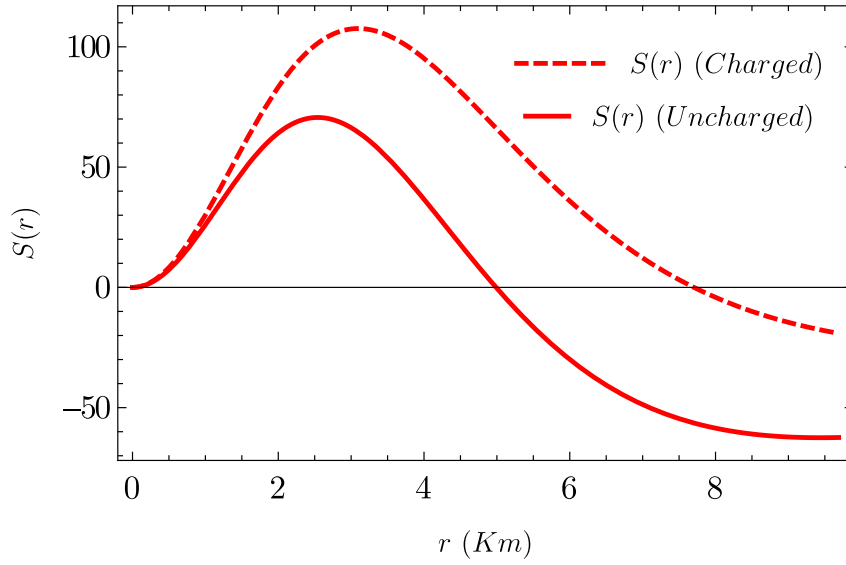


Fig. 4.— Variation of anisotropies against radial variable r .

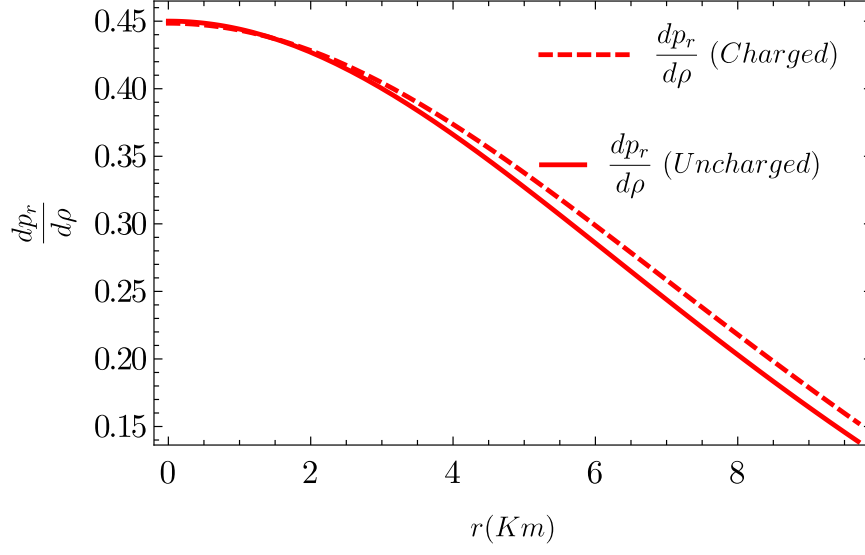


Fig. 5.— Variation of $\frac{1}{c^2} \frac{dp_r}{d\rho}$ against radial variable r .

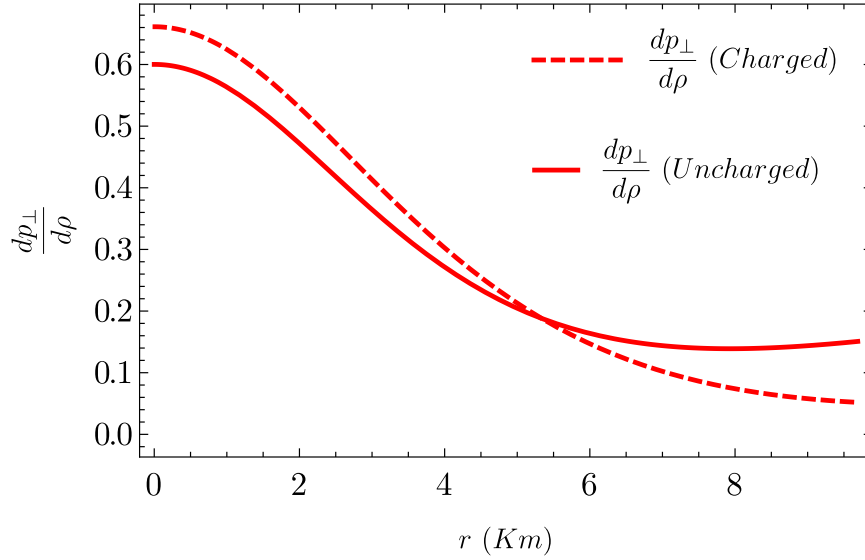


Fig. 6.— Variation of $\frac{1}{c^2} \frac{dp_{\perp}}{d\rho}$ against radial variable r .

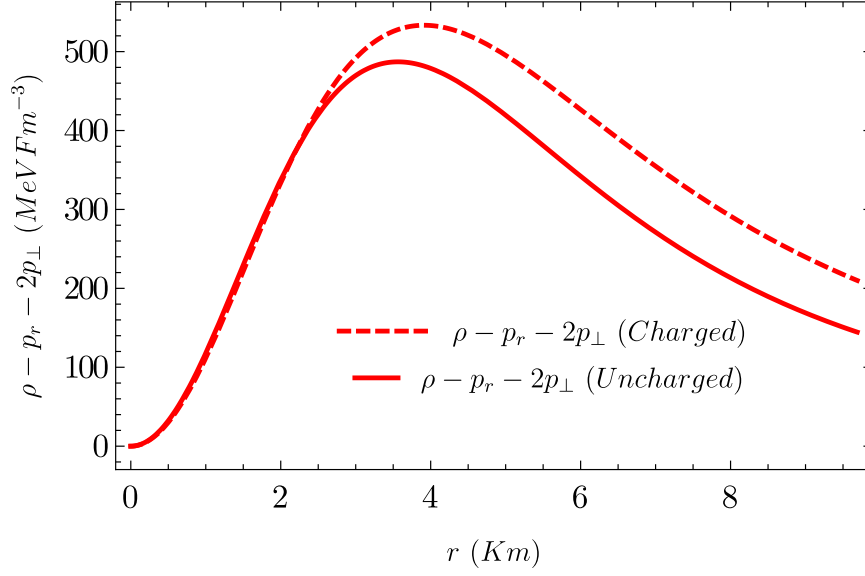


Fig. 7.— Variation of strong energy condition against radial variable r .

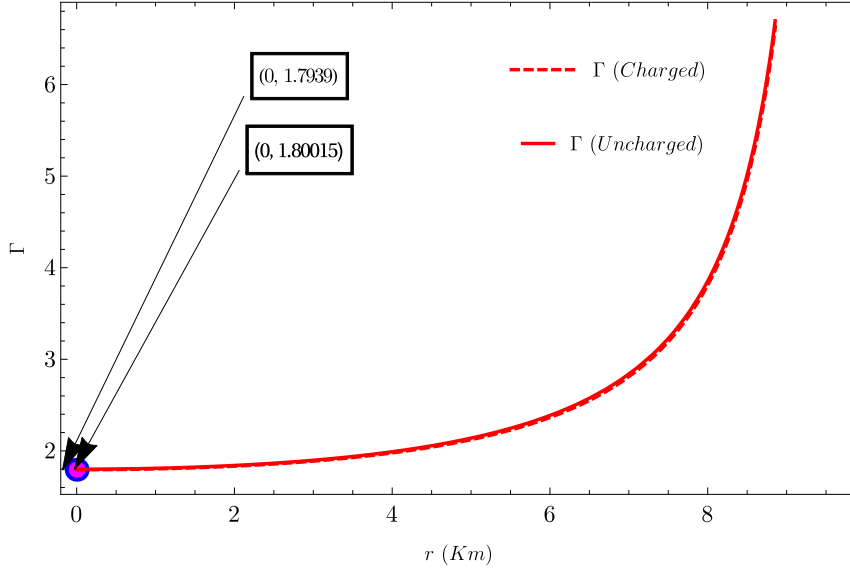


Fig. 8.— Variation of Γ against radial variable r .

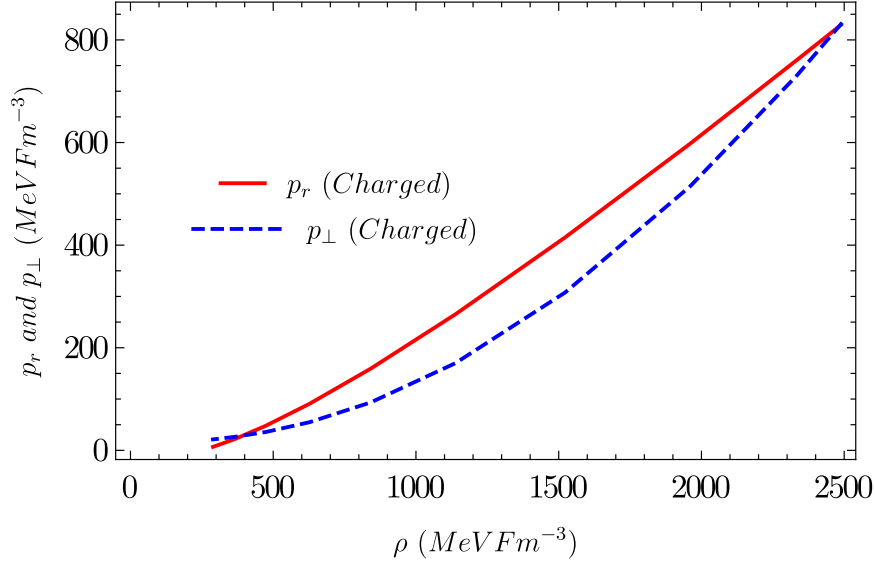


Fig. 9.— Variation of pressures against density for charged case.

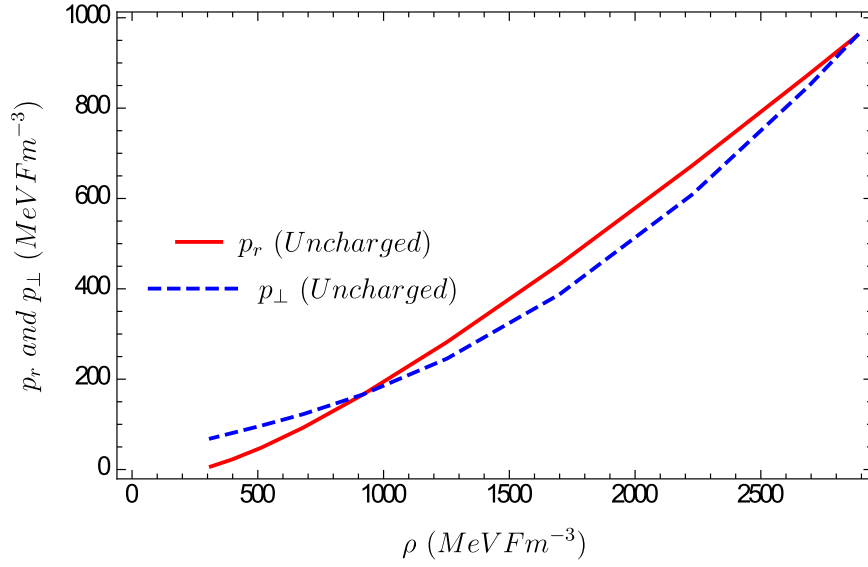


Fig. 10.— Variation of pressures against density for uncharged case.

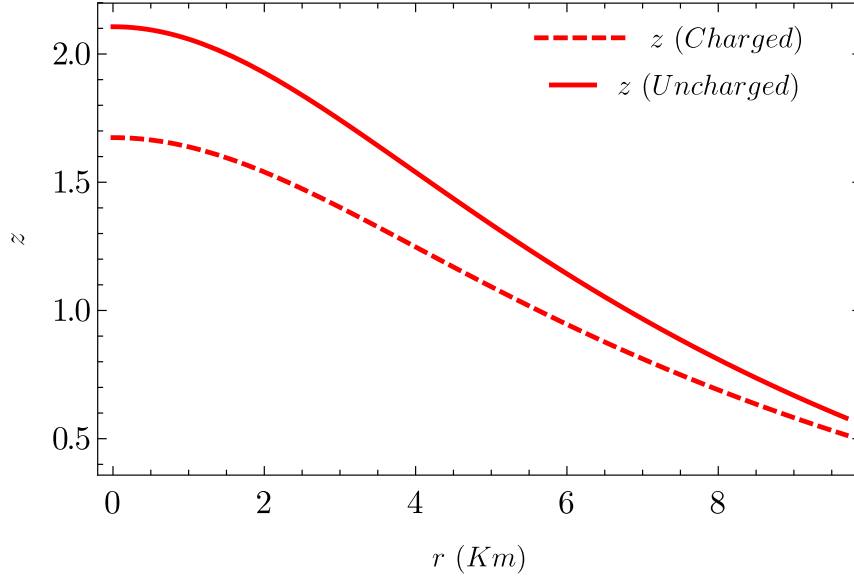


Fig. 11.— Variation of gravitational redshift against radial variable r .

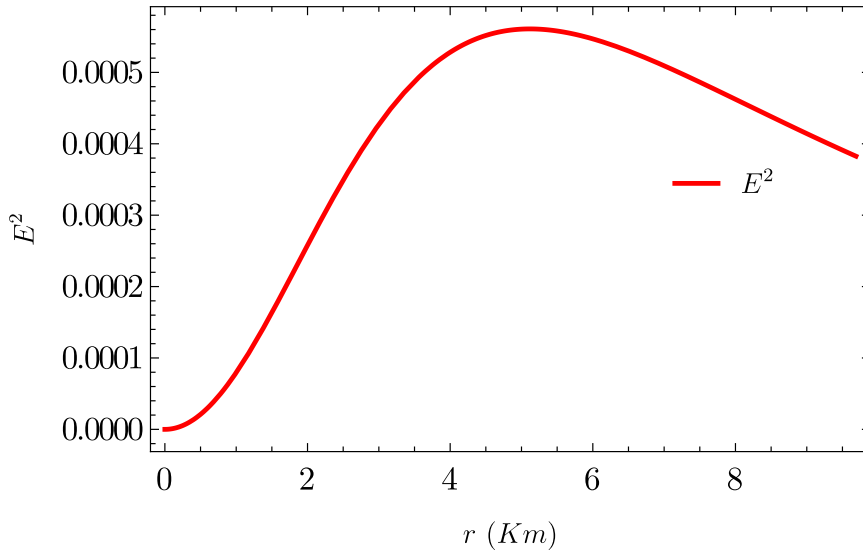


Fig. 12.— Variation of E^2 against radial variable r .

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